On Using Extreme Value Theory in Response-Time Analysis of Priority-Driven Periodic Real-Time Systems

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Abstract

In this paper we present work toward using our previously proposed method RapidRT to perform response-time analysis of periodic real-time systems, where the execution time of the adhering tasks is a random variable from a known distribution. In effect, we not only aim at validating the potential of considering the results given by RapidRT as upper bounds on tasks' worst-case response time estimates, but also investigate the possibility of using RapidRT as a good substitute for the referenced exact stochastic analysis method which is generally intractable for large systems.

1 Introduction

Traditional schedulability analysis methods for hard real-time systems are often based on a periodic task model, where the assumption on tasks' Worst-Case Execution Time (WCET) is made, in order to provide a deterministic and stringent guarantee that all the tasks in the system can meet their deadlines. If deadlines of all tasks are met, then the system is deemed schedulable. However, in the context of timing analysis for soft real-time systems where a failure in meeting timing requirements will not result in failure of system that potentially results in catastrophic human consequences, such a stringent guarantee is not required. Moreover, it is often better off providing a probabilistic guarantee that the deadline miss ratio of a task is below a certain threshold. Consequently, the assumption on tasks' WCET has to be relaxed.

A stochastic analysis framework is presented in [1], which does not introduce any worst-case or restrictive assumptions into the analysis, and is applicable to general priority-based real-time systems including both fixed-priority scheduling systems and dynamic-priority scheduling systems. The analysis method can handle any periodic task set consisting of tasks, each of which the execution time of the adhering jobs is specified as a random variable with a known discrete distribution. Furthermore, the analysis can give the exact *Probability Mass Function* (PMF) of response time of the tasks in the system, and a probability of deadline miss of tasks.

In [2], we present the timing analysis method namely *RapidRT*, which combines Extreme Value Theory (EVT) [3] with other statistical methods in order to produce a Worst-Case Response Time (WCRT) estimate of tasks, under a certain statistic constraint, i.e., a certain probability of being exceeded. Furthermore, RapidRT performs WCRT analysis of the target system based on a number of traces[1] containing Response Time (RT) data of tasks. In this work, we are interested in validating RapidRT, by examining if the results given by RapidRT can be considered as safe upper bounds on the WCRT estimates of tasks in the priority-driven periodic real-time systems [1]. In such systems, the exact WCRT values can be obtained through the stochastic analysis framework in [1]. Moreover, such work is meaningful and valuable in the sense that it can also help us to determine that, if RapidRT can be a substitute for the referenced stochastic analysis framework which is generally intractable for large systems.

2 System Model and Notations

The system model S consists of a set of n independent periodic tasks running on a uniprocessor, i.e., $S \leftarrow \tau_1, ..., \tau_n$, where $n \in \mathbb{N}$. A task τ_i is characterized by a set of parameters $\langle T_i, \Phi_i, e_i, D_i, M_i \rangle$, where T_i is the task period, Φ_i is the initial phase, e_i is execution time, D_i is the relative deadline or the temporal constraint, M_i is the maximum allowed probability of missing the deadline. The execution time e_i is a discrete random variable with a known distribution. The PMF is denoted by $f_{e_i}(\cdot)$, where $f_{e_i}(e) = P\{e_i = e\}$. Each periodic task results in an infi-

[1] Such traces are either from simulation models or the execution of the real system.

nite number of jobs. $\Gamma_{i,j}$ denotes the *j*th job of the task τ_i. Each job $\Gamma_{i,j}$ is released at a deterministic time $\lambda_{i,j}$, which is computed via Equation 1.

$$\lambda_{i,j} = \phi_i + (j-1)T_i \tag{1}$$

The response time of a job $\Gamma_{i,j}$ is a discrete random variable denoted by $R_{i,j}$, whereas the response time of a task R_i is computed by averaging the response time of its jobs as shown in Equation 2:

$$f_{R_i}(r) = \frac{1}{m_i}\sum_{j=1}^{m_i} f_{R_{i,j}}(r) \tag{2}$$

where $m_i = T/T_i$, which is the number of jobs from τ_i released in a hyper-period of length T. In addition, a task τ_i is said to be schedulable if $P\{R_i > D_i\} \leq M_i$.

3 The Stochastic Analysis Framework

The referenced stochastic analysis framework in this work, is proposed in [1], which will be summarized briefly as follows. For the sake of simplicity, the task to which a job belongs is not tracked, thus a job has a single index, e.g., Γ_j. The index of a job refers to its order in the infinite sequence of jobs, i.e., Γ_k is released before Γ_{k+1}, that is $\forall k$, $\lambda_k \leq \lambda_{k+1}$. The response time of a job Γ_j is computed as follows:

$$R_j = W(\lambda_j) + e_j + J_j \tag{3}$$

where R_j denotes the response time distribution of an arbitrary job Γ_j; $W(\lambda_j)$ denotes the backlog at time λ_j, i.e., the sum of the remaining execution times of all the jobs that do not finish up to time λ_j while having higher priorities than the job under analysis; J_j denotes the interference of all higher priority jobs released after job Γ_j.

The backlog at the release time of any job Γ_j, denoted by W_{λ_j}, can be computed by using the following iterative procedure [1]:

$$W(\lambda_{k_0}) = 0 \tag{4}$$

$$W(\lambda_k) = shrink(W(\lambda_{k-1}) + e_{k-1}, \lambda_k - \lambda_{k-1}) \tag{5}$$

where λ_{k_0} denotes the release time of the first job released before Γ_j and has a higher priority. The *shrink function* is given by:

$$f_{shrink}(W,\Delta)(x) = \begin{cases} 0 & \textit{if } x < 0, \\ \displaystyle\sum_{z=-\infty}^{0} f_W(z+\Delta) & \textit{if } x = 0, \\ f_W(x+\Delta) & \textit{if } x > 0. \end{cases} \tag{6}$$

Iterations start with a zero backlog as shown by Equation 4 and iterates on all higher priority jobs released before Γ_j. After computing the backlog at the release time of Γ_j, the backlog distribution is convolved with the execution time distribution. Such convolution results in a partial response time which is valid only if no interference with subsequent higher priority jobs takes place. In case of the existence of higher priority jobs released after λ_j, this partial response time will be valid only from λ_j to λ_{j+1}. A validity range is indicated as a super index for the response time $R^{[0,\lambda_{j+1}-\lambda_j]}$, which is computed as follows:

$$R^{[0,\lambda_{j+1}-\lambda_j]} = W(\lambda_j) + e_j \tag{7}$$

In order to increase the range of validity, the following equation is used [1]:

$$R^{[0,\lambda_{k+1}-\lambda_j]} = AF(R^{[0,\lambda_k-\lambda_j]}, \lambda_k - \lambda_j, e_k), k > j \tag{8}$$

where the job Γ_k has a higher priority than Γ_j, and AF is a stochastic function given by:

$$f_{AF(R,\Delta,e)}(x) = \begin{cases} f_R(x) & \textit{if } x \leq \Delta, \\ \displaystyle\sum_{i=\Delta+1}^{\infty} f_R(i)\cdot f_e(x-i) & \textit{if } x > \Delta. \end{cases} \tag{9}$$

Each iteration using Equation 8 increases the interval of validity of the partial response time. The iteration stops when the deadline is included in the validity range. Consequently, the probability of missing the deadline for a certain job can be computed by summing the probabilities of the response time values lying before the deadline and subtracting this sum from one:

$$P(R_j > D_j) = 1 - \sum_{k=0}^{k=D_j} P(R_j^{[0,\Delta]} = k) \tag{10}$$

After completing the analysis, the probability of missing the deadline of a certain task is computed by averaging the probabilities of missing the deadlines of all its jobs, as shown by Equation 2.

4 RapidRT Using Extreme Value Theory

Our proposed method RapidRT is based on Extreme Value Thoery (EVT) [3], which is a separate branch of statistics for dealing with the tail behavior of a distribution. EVT is used to model the risk of the extreme, rare events, without the vast amount of sample data required by a brute-force approach. Example applications of EVT include risk management, insurance, telecommunications and so on.

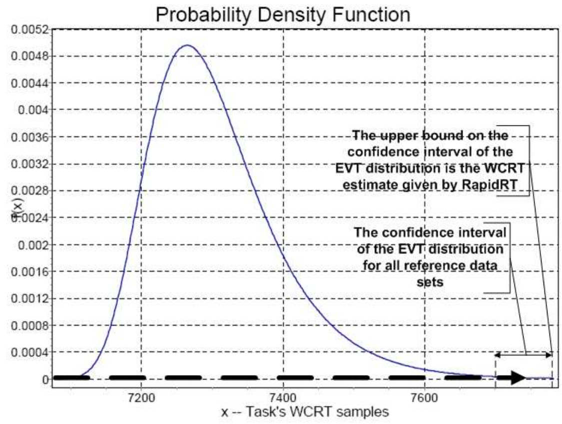

Figure 1. The point, at which the bold dash line intersects with the Gumbel Max curve, is the WCRT estimate given by RapidRT for each reference data set. The EVT distribution is constructed on these points for all reference data sets.

RapidRT is a recursive procedure which, as the first two arguments, takes n reference data sets each of which contains m simulation traces containing tasks' response times. For each reference data set, the algorithm returns the WCRT estimate of the task under analysis with a probability of being exceeded, e.g., 10^{-9}, which is the third algorithm argument. For instance, Airbus uses such the value 10^{-9} which is at the highest development assurance level in the safety-critical system domain. Next, RapidRT will verify if the sampling distribution consisting of n WCRT estimates given by EVT for all n reference data sets (we refer to such a sampling distribution as the EVT distribution hereafter) conforms to a normal distribution or not, according to the result given by the non-parametric Kolmogorov-Smirnov test (the KS test hereafter). If it is, then RapidRT will calculate the confidence interval (i.e., CI hereafter) of the EVT distribution, at the given confidence level 99.7%, and choose the upper bound on the CI as the final WCRT estimate, as shown in Figure 1. This invents a new hard statistic constraint, i.e., from the statistical perspective, given the modeled system, the possibility of the existence of a higher WCRT estimate (i.e., the actual WCRT of the task on focus) than the WCRT estimate given by RapidRT is no more than 1.5×10^{-12} (i.e., $(100\% - 99.7\%)/2 \times 10^{-9}$). Otherwise, if the EVT distribution cannot be fitted to a normal distribution, a *resampling* statistic *bootstrap* will be adopted to obtain the upper bound on the CI of the EVT distribution.

RapidRT consists of the following three steps: 1) construction of the referenced data sets, 2) WCRT estimation of each referenced data set using EVT, and 3) derivation of a final WCRT estimate that is given by the algorithm. For more details and thorough explanations about each step in RapidRT, the interested readers can refer to [2]. In addition, the outline of the algorithm is as follows:

1. Construct n reference data sets for the WCRT estimates by running m Monte Carlo simulations for each reference data at first, and then choosing the highest maximum value of response time of the task under analysis in each simulation. Consequently, the sampling distribution of RT data per reference data set consists of the m highest maximum RT data of m simulations.

2. Perform the WCRT estimates on the task under analysis per each reference data set, i.e., est_i where $1 \leq i \leq n$.

3. After verifying if the EVT distribution (i.e., $EST \leftarrow est_1, ..., est_i, ..., est_n$) can successfully be fitted to a normal distribution by using the KS test, RapidRT will return a result, i.e., $\overline{EST} + 3\sigma_{EST}$ (the sum of the *mean* value and 3 *standard deviation* of EST at the confidence level 99.7%). Otherwise, the bootstrap test will be used in the context.

5 Evaluation

The target priority-based periodic real-time systems (introduced in Section 2) will be modeled and analyzed by using our RTSSim simulation framework [4]. RTSSim is quite similar to *ARTISST* [5] and *VirtualTime* [6], and allows for simulating job-level system models on a single processor. Further, RTSSim provides typical RTOS services to simulation model, such as Fixed-Priority Preemptive Scheduling (FPPS), intricate task execution dependencies on job-level including Inter-Process Communication (IPC) via message queues and synchronization (semaphores). The execution time of jobs can be modeled as a random variable with a specific type of distribution. All time-related operations in RTSSim, such as timeouts and activation of time-triggered tasks, are driven by the simulation clock, which makes the simulation result independent of process scheduling and performance of the analysis PC. The response time and execution time of tasks or jobs are measured whenever the scheduler is invoked, which happens for example at IPC, task or job switches, *EXECUTE* statements, operations on semaphores, task or job activations and when tasks or jobs end. This, together with the simulation clock behavior, guarantees that the measured response time and execution time are exact. In RTSSim, a task may not be released for execution until a certain non-negative time (i.e., the offset) has elapsed after the arrival of the activating event. Each task also has a period, a maximum arrival jitter, and a priority. Tasks with equal priorities are served on the first come first serve basis.

In addition, we will propose a number of evaluation frameworks from the following perspectives:

1. **Different statistical constraints in RapidRT:** In our evaluation, the probabilities of being exceeded in RapidRT can be relatively either low or high, when compared to the one that we used in the previous research, i.e., 10^{-9}. For example, such probabilities can be $10^{-3}, 10^{-6}, 10^{-12}, 10^{-20}$ etc. The intention is to evaluate that if the results given by RapidRT can successfully cover the exact value of WCRT of tasks, when different levels of statistical constraints are applied.
2. **Different confidence levels of the EVT distribution:** We also consider using different confidence levels in the EVT distribution in RapidRT, such as 95%, which is a typical value and based on preliminary assessments provides appropriate results.
3. **Scalability of RapidRT:** This can be done by creating independent "subsystems" where each subsystem is a complete model, i.e., a priority-driven periodic real-time systems (as introduced in Section 2). For more details of using "subsystems" for scalability evaluation can be found in [7].
4. **Optimization on the number of samples in RapidRT:** The KS test will be used in this context with the purpose of optimizing the number of samples in RapidRT, while keeping the accuracy of results. This will reduce the computation time required by RapidRT, which is especially meaningful and necessary for the cases about timing analysis of large systems.

6 Related Work

As introduced in [1], the exact stochastic analysis of most real-time systems under preemptive priority driven scheduling is not affordable in practice currently. Some approaches about performing stochastic analysis with a specific scheduling model that isolates tasks so that each task can be analyzed independently are proposed [8, 9]. In addition, in order to simply the stochastic analysis in such context, the worst-case assumptions are introduced. Manolache presents the way of restricting tasks preemption, and some others [10, 11] introduce the assumption on the critical instance. In [12], Díaz furthers their previous study by introducing an approximate analysis, in order to decrease the memory demand on the computation of backlog and response time distributions. Recently, Refaat [13] proposes a method for efficient stochastic analysis by simplifying the exact distributions of jobs through random sampling.

7 Future Work

This work-in-progress paper has presented ongoing work on using our previously proposed method RapidRT in response-time analysis of priority-driven periodic real-time systems. We are also interested in using RapidRT as a good substitute for the referenced exact stochastic analysis method which is generally intractable for large systems. In particular, we have expressed the idea of comparing the results given by RapidRT to the ones obtained through the referenced stochastic analysis framework, which provides us with exact PMFs for task response times. Future work will mainly lie in implementation and evaluation.

References

[1] J. M. López, J. L. Díaz, J. Entrialgo, and D. García, "Stochastic analysis of real-time systems under preemptive priority-driven scheduling," *Real-Time Syst.*, vol. 40, no. 2, pp. 180–207, 2008.

[2] Y. Lu, T. Nolte, J. Kraft, and C. Norström, "A Statistical Approach to Response-Time Analysis of Complex Embedded Real-Time Systems," in *Proceedings of RTCSA' 10*, August 2010.

[3] J. Beirlant, Y. Goegebeur, J. Segers, and J. Teugels, *Statistics of Extremes: Theory and Applications*. Wiley Press, 2004.

[4] J. Kraft, "RTSSim – A Simulation Framework for Complex Embedded Systems," Mälardalen University, Tech. Report., Mar. 2009.

[5] D. Decotigny and I. Puaut, "ARTISST: An Extensible and Modular Simulation Tool for Real-Time Systems," in *Proceedings of ISORC' 02*, 2002, pp. 365–372.

[6] "Website of Rapita systems," 2008. [Online]. Available: www.rapitasystems.com

[7] M. Bohlin, Y. Lu, J. Kraft, P. Kreuger, and T. Nolte, "Simulation-Based Timing Analysis of Complex Real-Time Systems," in *Proceedings of RTCSA' 09*, August 2009, pp. 321–328.

[8] A. Atlas and A. Bestavros, "Statistical rate monotonic scheduling," in *Proceedings of RTSS' 98*, 1998, pp.123–132.

[9] G. A. Kaczynski, L. L. Bello, and T. Nolte, "Deriving Exact Stochastic Response Times of Periodic Tasks in Hybrid Priority-driven Soft Real-time Systems," in *Proceedings of ETFA' 07*, September 2007, pp. 101–110.

[10] T.-S. Tia, Z. Deng, M. Shankar, M. Storch, J. Sun, L.-C. Wu, and J. W.-S. Liu, "Probabilistic performance guarantee for real-time tasks with varying computation times," in *Proceedings of RTAS' 95*, 1995, pp. 164–173.

[11] M. K. Gardner and J. W.-S. Liu, "Analyzing stochastic fixed-priority real-time systems," in *Proceedings of TACAS' 99*, 1999, pp. 44–58.

[12] J. L. Diaz, J. M. Lopez, M. Garcia, A. M. Campos, K. Kim, and L. Lo Bello, "Pessimism in the stochastic analysis of real-time systems: Concept and applications," in *Proceedings of RTSS' 04*, 2004, pp. 197–207.

[13] K. S. Refaat and P.-E. Hladik, "Efficient stochastic analysis of real-time systems via random sampling," in *Proceedings of ECRTS' 10*, Brussels, Belgium, 2010, pp. 175–183.