

# Three-step Two-way Decode and Forward Relay with Energy Harvesting

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**Abstract**—Radio Frequency (RF) Energy Harvesting is being considered for realizing energy efficient relay networks. This work focuses on decode-and-forward relaying in an energy harvesting network and develops analytical expressions of the outage probability and overall throughput. A three-step scheme has been proposed that allows bidirectional exchange of information between two nodes via an intermediate relay. The performance of the proposed scheme has been evaluated and compared with a recent work.

**Index Terms**—Energy harvesting, decode-and-forward, two-way relay network, performance analysis.

## I. INTRODUCTION

Radio frequency based energy harvesting (RFEH) has received considerable attention as an effective approach for powering up the wireless nodes in future networks [1], [2]. RFEH relates particularly well with the cooperative networks, in which several relays are employed to extend wireless transmission range. A typical relay node allows exchange of information between two out-of-range nodes using either one-way or two-way schemes. One-way relaying allows information to transfer in one direction from the source through the relay to the destination. On the other hand, in two-way relaying, both nodes send information to the relay over a shared half-duplex channel [1]. This kind of relaying offers a more efficient use of the available resources. Two-way relaying can be performed in three steps (two slots for uplink and one for downlink) or even in two steps (one slot each for uplink and downlink) [3]. This paper considers three-step two-way relaying because it requires a relatively simpler circuit design. Most relay networks employ one of the two basic protocols: amplify and forward (AF) and decode and forward (DF), which have been evaluated in various previous works. For example, the authors in [3] have analyzed AF, joint decode-and-forward (JDF), DF and denoise-and-forward (DNF) protocols in terms of the maximal rate. It has been shown that DNF relaying outperforms the rest but at the same time uses disparate method when different modulation and coding mechanisms are used. The DNF protocol is also explored by Xu et al. in [4] using two-step relaying. A two-step relay mechanism results in multiple-access interference if the same uplink frequency is used by both nodes. On the other hand, the network compromises on spectral efficiency if different frequencies are assigned for the two nodes. In [5], the relay adds and forwards the two signals intended for two distinct destinations from the same source. Shengkai Xu et al. [6] have proposed a three-step two-way network using the product relay, in which the relay: (1) multiplies the received signals, (2) amplifies the result, and (3) forwards it to both nodes. Unlike

most previous works, Chen et al. [7] and Shah et al. [8] have introduced energy harvesting in their relay. The received power is split at the relay for performing two main tasks: information processing and data forwarding (using the harvested energy). The so-called power splitting factor determines the percentage of the received power dedicated for harvesting energy task. The overall throughput attained in [8] (use multiplicative and forward) is considerably larger than that obtained in [7]. In this paper, we consider two-way three-step DF relaying with energy harvesting and derive an expression for its signal-to-noise ratio (SNR). The motivation of using the DF relay comes from the facts that (1) very little is known in literature about the DF relays that use energy harvesting, and (2) the DF relay is found to be of more practical interest [9]. The rest of this paper is organized as follows. Section II presents the system model, underlying assumptions and problem statement. The analytical expressions of the lower and upper bounds of the outage probability and throughput are derived in Section III. The performance evaluation is reported in Section IV, and this paper is concluded in Section V.

## II. SYSTEM MODEL

Two nodes A and B exchange information via the relay node R as shown in the Fig. 1. The distance between A and B is such that direct transmission is not possible between them. Channels are assumed to be constant over the transmission block  $T$  and all channels are assumed to be reciprocal. The time block  $T$  is divided into three time slots in which  $\rho$  is the time proportion for the relay  $R$  to harvest energy and decode signal from one node ( $0 < \rho < 0.5$ ). In the first time slot  $t_1$ , the relay  $R$  receives signal from the node A, and it uses its power splitter to divide the signal power into two parts: one for harvesting energy and the other for processing signal (see Fig.2). In the second time slot  $t_2$ , the relay R repeats this process for node B with  $t_1 = t_2 = T\rho$ . Finally, in the third time slot  $t_3 = T(1 - 2\rho)$ , the relay  $R$  forwards its signal to the node A and B. The distances and the channel coefficients from R to

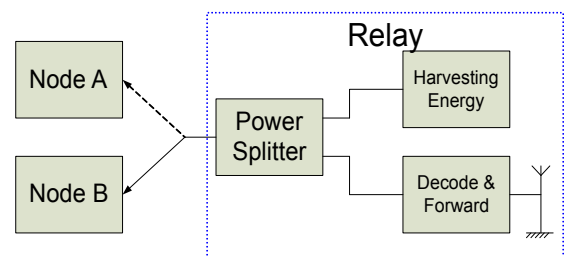


Fig. 1. Block diagram of the system

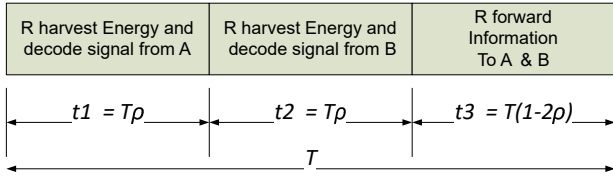


Fig. 2. Time block of three-step two-way relay

nodes A and B are  $d_A, d_B, g_A, g_B$  respectively. The frequency channels are independent and experience Rayleigh fading. The channel gains are exponentially distributed independent variables. The energy is harvested at the RF band while the detection takes place in the baseband. To implement RF energy harvesting mechanism in the practice, the harvester of devices should consist of a radio frequency (RF) to DC circuit converter, power management, and power storage parts. More specifically, the RF to the DC circuit is used to convert the RF signal into the DC voltage, while the power management regulates the output voltage of the storage device. The duty of the energy storage is to provide the voltage/energy for the system. The challenging to apply RF energy harvester is that the RF energy harvester behaves non-linearly with respect to the input power of the RF signal [10]. If the input power is low, the efficiency of the RF harvester reduces significantly while the needed energy for the processing signal at the base band does not change.

### III. PERFORMANCE ANALYSIS

#### A. Energy harvesting and information processing

1) *Energy harvesting*: The received signals at R during the time slot  $t_1 = t_2 = T\rho$  is given as follows [6], [8].

$$Y_{i \rightarrow R} = \sqrt{\frac{P_i}{d_i^\alpha}} g_i x_i + n_i \quad (1)$$

where  $Y_{i \rightarrow R}$  ( $i = A, B$ ) are the received signals at R and  $P_i$  denotes the transmit power of node  $i$ . The distance, the channel coefficient and the path loss from node  $i$  to R are  $d_i, g_i$  and  $\alpha$ , respectively. The signal from node  $i, x_i$ , are of unit mean is received with the additive white Gaussian noise  $n_i$ . The power splitter divides the received signals into two parts:  $\sqrt{\lambda_i} Y_{i \rightarrow R}$  for harvesting energy and  $\sqrt{1 - \lambda_i} Y_{i \rightarrow R}$  for signal processing, depending on the value of the power splitting factor,  $\lambda_i$  corresponding with node  $i = A, B$  [1], [4]. The amount of energy harvested from node  $i$  is as follows

$$E_i = \lambda_i \eta \frac{P_i}{d_i^\alpha} |g_i|^2 T \rho \quad (2)$$

where  $\eta$  is the energy efficiency factor of the harvester. We assume that  $P = P_A = P_B$ . The total energy harvested at R is therefore:

$$E_{total} = \eta T \rho P \left( \lambda_A \frac{|g_A|^2}{d_A^\alpha} + \lambda_B \frac{|g_B|^2}{d_B^\alpha} \right) \quad (3)$$

2) *Information processing*: The following portions of the received signals given in (1) are used for decoding and forwarding.

$$Y_{R \text{ from } i} = \sqrt{1 - \lambda_i} \left( \sqrt{\frac{P_i}{d_i^\alpha}} g_i x_i + n_i \right) \quad (4)$$

During the first two time slots, R decodes signals  $\bar{x}_A$  and  $\bar{x}_B$  from A and B, respectively. During time slot  $t_3$ , R broadcasts the normalized signal,  $x_R$ , to the two destinations.

$$x_R = \frac{\bar{x}_A + \bar{x}_B}{\sqrt{2}} \quad (5)$$

Furthermore, at node B

$$Y_{R \rightarrow B} = \sqrt{\frac{P_R}{d_B^\alpha}} g_B x_R + n_B \quad (6)$$

Substituting (5) into (6), node B already knew its own information therefore it easily discards  $x_B$  to get  $x_A$  from  $x_R$ . Here, we assume that the channel state information (CSI) and other system parameters are available at all nodes. We can estimate the signal received at B (from A via R) as

$$\hat{x}_{A \rightarrow B} = \sqrt{\frac{P_R}{d_B^\alpha}} g_B \frac{\bar{x}_A}{\sqrt{2}} + \hat{n}_B \quad (7)$$

where the noise at the node B has zero-mean and covariance  $\sigma_B^2$  and defined as  $\hat{n}_B = N(0, \sigma_B^2)$ . We assume that  $\lambda_A = \lambda_B = \lambda$ . However, it is important to note that if the relay uses only one power splitter for both links, then  $\lambda_A$  being different from  $\lambda_B$ , means that the power splitter should be tunable or adaptive which is complicated and expensive to implement. Hence, we consider using a simple power splitter with a fixed power splitting ratio  $\lambda$ . The power of R,  $P_R$ , can be expressed as

$$P_R = \frac{E_{total}}{T(1-2\rho)} = \eta \lambda P \left( \frac{|g_A|^2}{d_A^\alpha} + \frac{|g_B|^2}{d_B^\alpha} \right) \left( \frac{\rho}{1-2\rho} \right) \quad (8)$$

3) *Outage probability and throughput*: We first consider the signal sent from node A to B through the relay R. The SNR of this signal at R,  $\bar{\gamma}_R$ , can be calculated from (4) as

$$\bar{\gamma}_R = \frac{P |g_A|^2 (1 - \lambda)}{d_A^\alpha \sigma_A^2} \quad (9)$$

In which  $\sigma_A^2$  is variance of Gaussian noise at node A. Once the relay R forwards its signal to the node B, the SNR  $\bar{\gamma}_B$  is calculated as

$$\bar{\gamma}_B = \frac{P_R |g_B|^2}{d_B^2 \sigma_B^2} = \frac{(|g_A|^2 d_B^\alpha + |g_B|^2 d_A^\alpha) |g_B|^2}{b} \quad (10)$$

where  $b = \frac{d_B^{2\alpha} d_A^{2\alpha} 2\sigma_B^2}{\eta \lambda P} \frac{1-2\rho}{\rho}$ . Note from (9) that  $|g_A|^2$  has exponential distribution therefore  $\bar{\gamma}_R$  also follows the exponential distribution. Outage probability at the relay,  $P_{outR}$ , is defined as the probability that the SNR is dropped below a predefined threshold  $\gamma_{th}$ , given as

$$P_{outR} = \Pr\{\bar{\gamma}_R \leq \gamma_{th}\} = \Pr\left\{|g_A|^2 \leq \frac{\gamma_{th} d_A^\alpha \sigma_A^2}{(1-\lambda)P}\right\} \quad (11)$$

$$= 1 - e^{-\frac{\gamma_{th} d_A^\alpha \sigma_A^2}{(1-\lambda)P}}$$

Similarly, we can derive the outage probability at the node B,  $P_{out_B}$ , as follows

$$P_{out_B} = \Pr \{ \bar{\gamma}_B \leq \gamma_{th} \} = F_{\bar{\gamma}_B}(\gamma_{th}) \quad (12)$$

where the CDF of  $\bar{\gamma}_B$ ,  $F_{\bar{\gamma}_B}$ , is expressed as

$$F_{\bar{\gamma}_B}(\gamma) = \Pr \left\{ |g_A|^2 d_B^\alpha \leq \frac{\gamma b}{|g_B|^2} - |g_B|^2 d_A^\alpha \right\} \quad (13)$$

where  $\mu_A$  and  $\mu_B$  are the mean values of  $|g_A|^2$  and  $|g_B|^2$ , respectively.

Using the probability condition, we can derive  $F_{\bar{\gamma}_B}(\gamma)$  as follows

$$F_{\bar{\gamma}_B}(\gamma) = \int_0^\infty \Pr \left\{ |g_A|^2 \leq \frac{1}{d_B^\alpha} \left( \frac{\gamma b}{x} - x d_A^\alpha \right) \right\} f_{|g_B|^2}(x) dx$$

where  $f_{|g_B|^2}(x) = \frac{1}{\mu_B} e^{-\frac{x}{\mu_B}}$ .

Since  $|g_A|^2$  is an exponential random variable with mean value  $\mu_A$ , we have

$$F_{\bar{\gamma}_B}(\gamma) = \int_0^{x_{th}} \left( 1 - e^{-\left( \frac{\gamma b}{d_B^\alpha \mu_A} - x d_A^\alpha \right)} \right) \frac{1}{\mu_B} e^{-\frac{x}{\mu_B}} dx \quad (14)$$

$0 < x < x_{th} = \sqrt{\frac{\gamma b}{d_A^\alpha}}$  is the condition to valid  $\Pr \left\{ |g_A|^2 \leq \frac{1}{d_B^\alpha} \left( \frac{\gamma b}{x} - x d_A^\alpha \right) \right\}$ . By setting  $V = \frac{1}{\mu_B} - \frac{d_A^\alpha}{d_B^\alpha \mu_A}$ ,  $U = \frac{\gamma b}{d_B^\alpha \mu_A}$ , (14) can be rewritten as follows

$$F_{\bar{\gamma}_B}(\gamma) = 1 - e^{-\frac{x_{th}}{\mu_B}} - \frac{1}{\mu_B} \int_0^{x_{th}} e^{-\frac{U}{x} - Vx} dx \quad (15)$$

It is noted that there is no closed-form expression for (15), however, the  $F_{\bar{\gamma}_B}(\gamma)$  can be calculated by using popular numerical software such as Mathematica. Instead of finding closed-form expression for  $F_{\bar{\gamma}_B}(\gamma)$ , we derive the bounds by using a fact that  $2 \min\{Y_1, Y_2\} \leq Y_1 + Y_2 \leq 2 \max\{Y_1, Y_2\}$  with  $Y_1, Y_2 \geq 0$ . Accordingly, we have  $2 \min\{Y_1, Y_2\} - Y_3 \leq Y_1 + Y_2 - Y_3 \leq 2 \max\{Y_1, Y_2\} - Y_3$  with  $Y_3 > 0$ . As a result, we have  $\Pr\{2 \min\{Y_1, Y_2\} - Y_3 < 0\} \geq \Pr\{Y_1 + Y_2 - Y_3 < 0\} \geq \Pr\{2 \max\{Y_1, Y_2\} - Y_3 < 0\}$ . In other word, we obtain the upper bound and lower bound as follows:

$$\Pr\{Y_1 + Y_2 < Y_3\} \leq \Pr\{2 \min(Y_1, Y_2) < Y_3\} \quad (16)$$

$$\Pr\{2 \max(Y_1, Y_2) < Y_3\} \leq \Pr\{Y_1 + Y_2 < Y_3\} \quad (17)$$

Accordingly, the bound of  $F_{\bar{\gamma}_B}(\gamma)$  can be expressed as:

$$P_1(\gamma) \leq \Pr \left\{ |g_A|^2 d_B^\alpha + |g_B|^2 d_A^\alpha \leq \frac{\gamma b}{|g_B|^2} \right\} \leq P_2(\gamma) \quad (18)$$

where  $P_1(\gamma)$  and  $P_2(\gamma)$  are expressed, respectively, as

$$P_1(\gamma) = \Pr \left\{ 2 \max(|g_A|^2 d_B^\alpha, |g_B|^2 d_A^\alpha) \leq \gamma b / |g_B|^2 \right\} \quad (19)$$

$$P_2(\gamma) = \Pr \left\{ 2 \min(|g_A|^2 d_B^\alpha, |g_B|^2 d_A^\alpha) \leq \gamma b / |g_B|^2 \right\} \quad (20)$$

$$\begin{aligned} P_1(\gamma) &= \int_0^\infty \Pr \left\{ x d_B^\alpha \leq \frac{\gamma b}{2|g_B|^2} \right\} \Pr \left\{ |g_B|^2 d_A^\alpha \leq \frac{\gamma b}{2|g_B|^2} \right\} \\ &\quad \times f_{|g_A|^2}(x) dx \\ &= \int_0^\infty \left( 1 - e^{-\frac{\gamma b}{2x d_B^\alpha \mu_B}} \right) \left( 1 - e^{-\sqrt{\frac{\gamma b}{2d_A^\alpha \mu_B^2}}} \right) \frac{1}{\lambda_A} e^{-\frac{x}{\lambda_A}} dx \end{aligned} \quad (21)$$

After simplification, we finally obtain  $P_1$  [10, Eq.(3.324.1)] as follows, where  $K_1(\cdot)$  is the first order modified Bessel function of the second kind [10]:

$$\begin{aligned} P_1(\gamma) &= \left[ 1 - \sqrt{\frac{2b\gamma}{d_B^\alpha \mu_A \mu_B}} K_1 \left( \sqrt{\frac{2b\gamma}{d_B^\alpha \mu_A \mu_B}} \right) \right] \\ &\quad \times \left[ 1 - e^{-\sqrt{\frac{\gamma b}{2d_A^\alpha \mu_B^2}}} \right] \end{aligned} \quad (22)$$

Similarly, the  $P_2$  can be expressed as

$$\begin{aligned} P_2(\gamma) &= \Pr \left\{ 2 \min(|g_A|^2 d_B^\alpha, |g_B|^2 d_A^\alpha) \leq \frac{\gamma b}{|g_B|^2} \right\} \\ &= \int_0^\infty \Pr \left\{ 2 \min(x d_B^\alpha, |g_B|^2 d_A^\alpha) \leq \frac{\gamma b}{|g_B|^2} \right\} f_{|g_A|^2}(x) dx \\ &= 1 - \int_0^\infty \left( e^{-\frac{\gamma b}{2x \mu_B d_B^\alpha}} \right) \left( e^{-\sqrt{\frac{\gamma b}{2d_A^\alpha \mu_B^2}}} \right) \lambda_A e^{-\frac{x}{\lambda_A}} dx \end{aligned} \quad (23)$$

After several mathematical manipulations,  $P_2$  is given as.

$$P_2(\gamma) = 1 - e^{-\sqrt{\frac{\gamma b}{2d_A^\alpha \mu_B^2}}} \sqrt{\frac{2\gamma b}{d_B^\alpha \mu_A \mu_B}} K_1 \left( \sqrt{\frac{2\gamma b}{d_B^\alpha \mu_A \mu_B}} \right) \quad (24)$$

From (22) and (24), the boundary of  $P_{out_B}$  can be estimated as follows:

$$P_1(\gamma_{th}) \leq P_{out_B} \leq P_2(\gamma_{th}) \quad (25)$$

The link A to R and R to B are independent to the end-to-end throughput of the considered system is:

$$T_{E2E} = (1 - P_{out_B})(1 - P_{out_R})U\rho \quad (26)$$

where  $U$  is the source transmission rate of nodes A, B and R.

#### IV. NUMERICAL RESULTS

In this section, we study the impact of (i)  $\lambda$  and (ii) distances  $d_A, d_B$  on the system throughput. We examine the throughput when  $d_A = d_B = 1$ , and also when  $d_A$  is different from  $d_B$  but  $d_A + d_B = 2$ . The transmission rate is assumed to be  $U = 3$  bits/secs/Hz with the transmit power of the source node set to 1.5 Joules/sec. The threshold SNR  $\gamma_{th} = 2^U - 1$ , the energy coefficient  $\eta = 1$  and path loss coefficient  $\alpha = 2.7$ . The noise variance at all nodes is 0.01 and  $\rho = 1/3$ . For fair comparison, all parameters are set to the values used [8]. The simulation results are based on equations (11) and (12), and analytical results use equations(11) and (25). In practical terms, the received signal strength affects the efficiency of the harvester. For example, the efficiency of the harvester is 77.8 % when the receiving power is 10 dBm, which degrades to

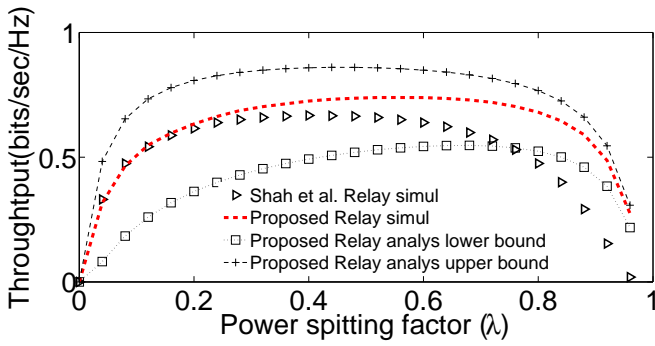


Fig. 3. The throughput versus the power splitting factor  $\lambda$  when  $d_A = d_B$ ,  $\mu_A = \mu_B$  the transmission rate  $U = 3$ .

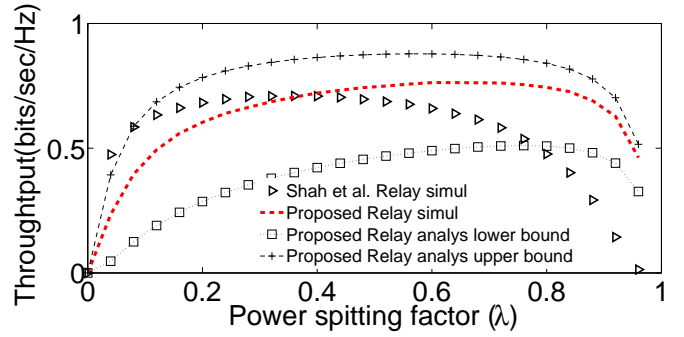


Fig. 5. The throughput versus the power splitting factor  $d_B/d_A = 1.5$ .

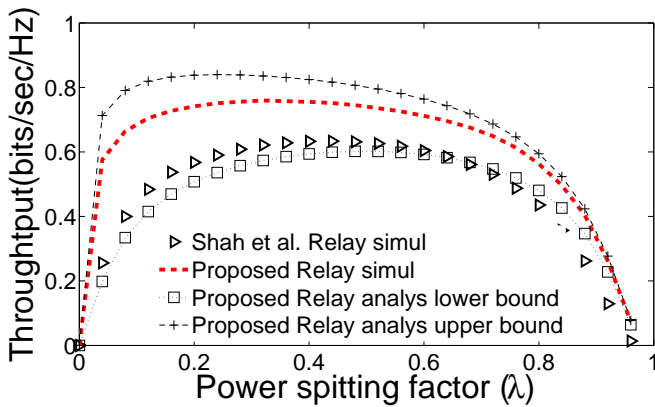


Fig. 4. The throughput versus the power splitting factor when  $d_B/d_A = 1/2$ , transmission rate  $U = 3$ .

28.4 % if the received power is  $-20$  dBm [11]. Consequently, when the distance between the nodes and the relay increases, the throughput of the system scales down.

Fig. 3 plots the system throughput as a function of  $\lambda$  assuming that  $d_A = d_B = 1$  (as in [8]). Fig. 3 plots the upper, lower bounds and simulation result of the proposed scheme together with Shah et al. result. The proposed system has a higher throughput than that reported in [8]. Fig. 3 clearly shows that the simulation results closely follow the analytical findings. Contrary to [8], we set  $d_B/d_A$  to 0.5, 1.5 and report the results in Fig. 4 and Fig. 5. It can be seen from the figures that when node  $B$  is closer to  $R$ , the overall throughput of the proposed model is still better than Shah et al. [8]. In the range of  $\lambda$  lower than 0.5 throughput displays the superior than the remaining range. Contrarily, our proposed scheme (fig 5) results in a lower throughput compared to that of [8] when  $\lambda < 0.4$ . In Fig. 5, the maximum throughput of [8] is comparable to the maximum throughput of proposed scheme but it obtains at different power splitter factor ( $\lambda = 0.2$  and  $\lambda = 0.8$ )

## V. CONCLUSION

This paper has proposed a two-way DF relay that exchanges information between a pair of nodes in three steps. The performance of the proposed relay is compared with a work on multiplicative AF relay. The expressions for the outage

probability of the proposed relay, and its upper and lower bounds have been evaluated. The simulation and analytical results show that the proposed relay outperforms multiplicative relay when the nodes are equidistant from the relay. However, when the distances are unequal, the performance of the proposed relay depends on the power splitting factor.

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